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ABSTRACTS

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MEASURES OF NONCOMPACTNESS IN MODULAR SPACES: FIXED POINT FOR SET-VALUED NON-EXPANSIVE MAPPINGS

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Abstract

In this talk we will discuss some fixed point results for multivalued mappings in modular vector spaces. For this purpose, we will consider measure of noncomapctness in modular spaces and we will study the uniform noncompact convexity, a geometric property for modular spaces which is similar to nearly uniform convexity in the Banach spaces setting.

COUPLED FIXED POINTS VIA SIMULATION FUNCTIONS

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Abstract

In this work, we present a result in complete metric spaces concerning the existence and uniqueness of coupled fixed points by means of simulation functions. These functions are employed in generalizations of the Banach contraction mapping principle, which is a fundamental tool in the theory of existence of solutions to functional, differential, and integral equations. Furthermore, as an application of our main result, we investigate the existence and uniqueness of solutions to a coupled system of functional equations.

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HOMOGENEOUS ORTHOGONAL POLYNOMIALS IN SEVERAL VARIABLES

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Abstract

We start by showing that the notion of homogeneous polynomials makes no sense in one variable case. It turns out that applying the approach developed in [2] one may find many examples of orthogonal polynomial systems in several variables satisfying the homogeneity requirement. This involves polynomials orthogonal on algebraic subsets of $\mathbb{R}^n$, a Euclidean sphere being one of them. In fact, the case of homogeneity is strictly related to the algebraic sets induced by quadrics. We will discuss this and some related questions.

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SOLVING QUADRATIC INTEGRAL EQUATIONS AND RELATED PROPERTIES OF MEASURES OF NONCOMPACTNESS

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Abstract

Quadratic integral equations, and more generally, quadratic operator equations, are an interesting research topic in mathematics, both because of their applications (cf. [3, 7, 1, 12]) and because of the differences in studying them compared to classical equations (cf. [2, 3, 4, 7, 10, 11]). One problem is applying methods based on fixed point theorems, especially Darbo's theorem, to these equations.

Here, our considerations begin with a series of results obtained by Banaś and his colleagues for Banach algebras ([1, 4, 5]). These results concern the study of realistic conditions with measures of noncompactness in such a space, which allows for the demonstration of the contraction condition of the quadratic operator with an appropriately chosen measure of noncompactness ([1, 2, 10]).

We will briefly outline the convincing reasons for such research. Building on the Maligranda-Orlicz lemma ([6]), we will extend existing results by demonstrating the relationship between norm selection on Banach algebras and the relevant properties of measures of noncompactness. This will be supplemented by remarks on quasi-normed algebras and analogous results for triples of spaces ([8]), so we go beyond the domain of Banach algebras.

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RECENT RESULTS ON MATRIX WEIGHTED NORM INEQUALITIES

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Abstract

Matrix weighted inequalities for singular integrals and other operators have been of interest since the work of Nazarov, Treil and Volberg in the 1900s. They asked the question: can we generalize the classical Muckenhoupt A_p condition to matrix weights so that we have inequalities of the form

$$\int_{\mathbb{R}^n} |W(x)Tf(x)|^p\,dx \leq C\int_{\mathbb{R}^n} |W(x)f(x)|^p\,dx,$$

$1 < p < \infty$, where W is a $d \times d$ self-adjoint, positive semi-definite matrix, f is an $\mathbb{R}^d$-valued measurable function, and T is a singular integral or other linear operator from classical harmonic analysis? NTV proved bounds for the Hilbert transform on the real line, introducing a generalization of A_p in terms of norm functions on $\mathbb{R}^d$. Their results were extend to $\mathbb{R}^n$ and general Calderón-Zygmund singular integrals by Christ and Goldberg. Later, Roudenko introduced an equivalent definition of matrix $\mathcal{A}_p$ that more closely resembles the classical scalar condition:

$$[W]_{\mathcal{A}_p} = \sup_Q \left(\fint_Q \left(\fint_Q |W(x)W^{-1}(y)|_{\text{op}}^{p'}\,dy \right)^{\frac{p}{p'}} dx \right)^{\frac{1}{p}} < \infty.$$

In the past 10 years there has been a great deal of research in the study of matrix weights. In this talk we will discuss recent work and open problems in this area, including: the sharp constant problem for singular integrals; Rubio de Francia extrapolation and the machinery of convex set-valued functions developed to prove this result; multiplier weak-type norm inequalities; and extensions to Banach functions spaces, particularly the variable Lebesgue spaces.

This talk will include work with my colleagues Marcin Bownik, Josh Isralowitz, and Kabe Moen, and with my PhD student Michael Penrod and my postdoc Fatih Şirin.

On the Generalized Grünbaum Conjecture

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Abstract

Let X be a Banach space over $\mathbb{K}$, where $\mathbb{K}=\mathbb{R}$ or $\mathbb{K}=\mathbb{C}$. Let $Y\subset X$ be a subspace. By $\mathcal{P}(X,Y)$ denote the set of all linear and continuous projections from X onto Y, recalling that an operator $P\colon X\to Y$ is called a *projection* onto Y if $P|_Y=\mathrm{Id}_Y$. We define the *relative projection constant* of subspace Y of space X by

$$\lambda(Y,X):=\inf\{\|P\|:\ P\in\mathcal{P}(X,Y)\}.$$

Now we can define the *absolute projection constant* of Y by

$$\lambda(Y):=\sup\{\lambda(Y,X):Y\subset X\}. \tag{0.1}$$

The ultimate goal of researchers in this area is to determine the exact value of *maximal absolute projection constant*, which is defined by

$$\lambda_{\mathbb{K}}(m):=\sup\{\lambda(Y):\ \dim(Y)=m\}.$$

In 1960, B.Grünbaum conjectured that $\lambda_{\mathbb{R}}(2)=\frac{4}{3}$ (see [4]), and in 2010, B. Chalmers and G. Lewicki proved it (see [3]). In 1994, H. König and N. Tomaczak-Jaegermann stated the following estimation, which is the generalization of the Grünbaum Conjecture

Theorem 1 (stated in [5]; proved in [1])**.** *Let $m>1$ then*

i) $\lambda_{\mathbb{R}}(m)\le\frac{2}{m+1}\left(1+\frac{m-1}{2}\sqrt{m+2}\right)$

ii) $\lambda_{\mathbb{C}}(m)\le\frac{1}{m}\left(1+(m-1)\sqrt{m+1}\right)$.

Furthermore, the inequality becomes an equality if there exists a maximal ETF in $\mathbb{K}^m$.

Unfortunately, their proof is based on an erroneous lemma, as was pointed out in [2]. In this talk, we will present the correct proof of the latter. Moreover, relying on this result, we provide the exact values of $\lambda_{\mathbb{K}}(m)$ in cases where a maximal equiangular tight frame exists in $\mathbb{K}^m$.

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The Banach–Mazur distance between isomorphic spaces of continuous functions is not always an integer

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Abstract

For over fifty years, Aleksander Pełczyński's question remained open: is the Banach–Mazur distance between any two isomorphic $C(K)$ spaces always an integer? In this talk, we provide a negative answer. We present an example of a pair of isomorphic $C(K)$ spaces whose Banach–Mazur distance is not an integer. Furthermore, we extend this result to a broader class of spaces by showing that the Banach–Mazur distance between $C(K)$ and $C_0(L)$, where L is a locally compact but noncompact Hausdorff space, can also be non-integer.

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THE MINIMAL DISPLACEMENT PROBLEM

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Abstract

Let B_X denote the closed unit ball in a Banach space X. The celebrated Brouwer's fixed point theorem states that every continuous self-map $T : B_X \to B_X$ has a fixed point, whenever X is finite dimensional. On the other hand, if space X is infinite dimensional, then it is always possible to construct a continuous self-map of B_X which is fixed point free. In the literature one can find many examples of uniformly continuous, lipschitzian and even nonexpansive maps $T : B_X \to B_X$ without fixed points. However, it is usually the case that the quantity

$$d(T) := \inf \{\|x - Tx\| : x \in B_X\},$$

which is called *the minimal displacement* of T, is zero. The question whether there exist lipschitzian self-maps of bounded, closed and convex sets with positive minimal displacement was settled affirmatively by Goebel in [1]. Ten years later Lin and Sternfeld [2] proved a remarkable theorem: if C is bounded, closed, convex and noncompact subset of an infinite dimensional Banach space, then for any $k > 1$ it is possible to construct k-Lipschitz map $T : C \to C$ with positive minimal displacement $d(T) > 0$.

Our goal for this presentation is to give an overview of some recent progress in the so called *minimal displacement problem*, originally introduced by K. Goebel in 1973, which involves identifying those Banach spaces which are *extremal* with respect to the minimal displacement of k-lipschitzian self-maps of the closed unit ball.

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FIXED POINT THEORY FOR BLOCK OPERATOR MATRICES

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Abstract

Many problems arising in mathematical physics, biology, etc., may be described in a first formulation, using systems of integral equations as well as systems of partial or ordinary differential equations. The theory of block operator matrices opens up a new line of attack of these problems. The aim of this talk is to discuss the existence of fixed points for a 2×2 block operator matrix L by laying down some conditions on the entries, which are generally nonlinear operators. This discussion is based on the presence or absence of invertibility of the diagonal terms of $I - L$. The results are applied to study the existence of systems of transport equations on L_p spaces. Due to the lack of compactness in L_1 spaces, the analysis is carried out via the arguments of weak topology and particularly the measure of weak noncompactness.

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EXISTENCE OF SOLUTIONS FOR INFINITE SYSTEMS OF DIFFERENTIAL EQUATIONS IN SPACES OF TEMPERED SEQUENCES

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Abstract

The goal of the paper is to investigate the existence of solutions for infinite systems of differential equations. We look for solutions in Banach tempered sequence spaces, using techniques associated with measures of noncompactness, and results from differential equations in abstract Banach spaces.

On the Link between Maximal Projection Constants and Spherical Designs

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Abstract

Let X be a Banach space over $\mathbb{K}$, where $\mathbb{K} = \mathbb{R}$ or $\mathbb{K} = \mathbb{C}$. Let $Y \subset X$ be a subspace. By $\mathcal{P}(X,Y)$ denote the set of all linear and continuous projections from X onto Y, recalling that an operator $P\colon X \to Y$ is called a *projection* onto Y if $P|_Y = \mathrm{Id}_Y$. We define the *relative projection constant* of a subspace Y of a space X by

$$\lambda(Y,X) := \inf\{\|P\| : P \in \mathcal{P}(X,Y)\}.$$

Now we can define the *absolute projection constant* of Y by

$$\lambda(Y) := \sup\{\lambda(Y,X) : Y \subset X\}. \tag{0.2}$$

The ultimate goal of researchers in this area is to determine the exact value of *maximal absolute projection constant*, which is defined by

$$\lambda_{\mathbb{K}}(m) := \sup\{\lambda(Y) : \dim(Y) = m\}.$$

In 1960, B.Grünbaum conjectured that $\lambda_{\mathbb{R}}(2) = \frac{4}{3}$ (see [6]), and only in 2010, B. Chalmers and G. Lewicki proved it (see [2]) and that was the only known nontrivial case. Recently, we have provided exact values of $\lambda_{\mathbb{K}}(m)$ in cases where the maximal equiangular tight frame exists in $\mathbb{K}^m$. There are numerous examples of complex maximal ETFs, for example, for $m \in \{1,\dots,17,19,24,28,35,48\}$ (see, e.g., [5]). In fact, it is conjectured that there is a complex maximal ETF in every dimension (Zaurner's conjecture [7]]). Unlike in the complex case, real maximal ETFs seem to be rare objects. The only known cases are for m equal to 2, 3, 7 and 23. A lot of the community believes that these are all real cases where maximal ETFs

exist. In other cases, the determination of the constant $\lambda_{\mathbb{R}}(m)$ seems to be difficult. Relying on the new construction of certain mutually unbiased equiangular tight frames, we showed that $\lambda(5) \geq 5(11+6\sqrt{5})/59 \approx 2.06919$ (see [3]). This value coincides with the numerical estimation of $\lambda(5)$ obtained by B. L. Chalmers, thus reinforcing the belief that this is the exact value of $\lambda(5)$. We briefly discuss the above results in the talk and present conjectures based on some new concepts.

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RECENT RESULTS ON MINIMAL PROJECTIONS

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Abstract

Let X be a Banach space and let $Y \subset X$ be its finite-dimensional subspace. Let $\mathcal{P}(X,Y)$ denote the set of all linear and continuous projections from X onto Y, recalling that an operator $P\colon X \to Y$ is called a *projection* onto Y if $P|_Y = \mathrm{Id}_Y$. We define the *relative projection constant* of Y by

$$\lambda_{\mathbb{K}}(Y,X) := \inf\{\|P\| :\ P \in \mathcal{P}(X,Y)\}$$

and the *absolute projection constant* of Y by

$$\lambda_{\mathbb{K}}(Y) := \sup\{\lambda(Y,X) : Y \subset X\}, \tag{0.3}$$

and finally the *maximal absolute projection constant*, by

$$\lambda_{\mathbb{K}}(m) := \sup\{\lambda(Y) :\ \dim(Y) = m\}.$$

The aim of my talk is to present recent results concerning the above defined constants obtained by me and my collegues from Kraków.

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De Branges-Rovnyak Spaces and Their Properties

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Abstract

De Branges-Rovnyak spaces are a class of reproducing kernel Hilbert spaces determined by contractive analytic functions. In this talk we present selected insights into properties of these spaces, with particular focus on generalizing known orthogonal decompositions to cases where the contractive function takes specific forms involving inner functions.

The talk is based on joint work with Małgorzata Michalska and Maria Nowak.

THE BANACH–MAZUR DISTANCE BETWEEN $C([1,\omega^n])$ AND $C([1,\omega])$

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Abstract

We give precise values of the Banach–Mazur distance between spaces $C([1,\omega^n])$ and $C([1,\omega])$ of all continuous real-valued functions on ordinal intervals endowed with the order topology. This solves a long-lasting problem posed by Bessaga and Pełczyński. Joint work with Łukasz Piasecki.

References

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BOUNDED LINEAR AND COMPACT OPERTORS ON THE GENERALISED HAHN SPACE

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Abstract

We present the characterisations of the classes of bounded linear operators from the generalised Hahn space into the classical sequence spaces, inequalities or identities for their operator norms, their subclasses of compact operators and inequalities or identities for their Hausdorff measures of noncompactess; we also list the corresponding results in the cases where the order of spaces is reversed ([1]). Furthermore, we list mention similar results where the classical sequence spaces are replaced by the sets of sequences that are strongly summable with index $p \geq 1$ by the Cesàro mehod of order 1 ([2, 3]). Finally, we give some applications of our results.

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SELECTED GEOMETRIC PROPERTIES OF INTERPOLATION SPACES

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Abstract

The aim of the presentation is to show results on geometric properties in interpolation spaces obtained with the help of an abstract discrete K-method based on a space with an unconditional basis. We present how geometric properties such like Opial properties and uniform convexity, uniform convexity in every direction and other behave after passing to interpolation spaces.

We say that the pair $\mathbf{X} = (X_0, X_1)$ is *an interpolation pair* if both those Banach spaces are subspaces of a common Hausdorff topological vector space V so that the embeddings of X_0 and of X_1 into V are continuous.

Definition 1. Let $1 \leq p < \infty$ be fixed. Let $\mathbf{X} = (X_0, X_1)$ be an interpolation pair. Let $(E, ||\cdot||_E)$ be a Banach lattice with a normalized unconditional basis $\{e_i\}_{i\in\mathbb{Z}}$ whose unconditional constant equals one. Let $\{a_i\}_{i\in\mathbb{Z}}$ and $\{b_i\}_{i\in\mathbb{Z}}$ be sequences of positive scalars for which $\sum_{ß\in\mathbb{Z}} \min(a_i, b_i) \leq \infty$. We define an *interpolation space* $K_p(\mathbf{X}, E, \{a_i\}, \{b_i\})$ as a space of all elements $x \in X_0 + X_1$ such that $\sum_{i\in\mathbb{Z}} k_p(x, a_i, b_i)\, e_i$ converges in E with the norm

$$||x||_{K_p} = \left|\left|\sum_{i\in\mathbb{Z}} k_p(x, a_i, b_i) e_i\right|\right|_E ,$$

where the space $Y_p = (X_0 + X_1, ||\cdot||_p)$ is defined as a space of all elements $x \in X_0 + X_1$ such that $x = x^0 + x^1$, where $x^0 \in X_0$ and $x^1 \in X_1$ and is equipped with the norm

$$||x||_p = k_p(x, a, b) = \inf\left\{ \left(a^p ||x^0||^p_{X_0} + b^p ||x^1||^p_{X_1}\right)^{\frac{1}{p}} : x = x^0 + x^1, x^0 \in X_0, x^1 \in X_1 \right\},$$

with $a, b \geq 0$.

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FIXED POINT THEORY IN WC AND RWC–BANACH ALGEBRAS

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Abstract

In this presentation, we establish several fixed point theorems for operators acting in WC and RWC–Banach algebras under assumptions involving generalized Lipschitz conditions and measures of weak noncompactness. Moreover, these assumptions are formulated using the weak topology and weak sequential continuity.

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DE BRANGES–ROVNYAK SPACES AND LOCAL DIRICHLET SPACES OF HIGHER ORDER

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Abstract

We study local Dirichlet spaces of order m introduced by S. Luo, C. Gu and S. Richter in [1] and de Branges–Rovnyak spaces $\mathcal{H}(b)$ generated by nonextreme and rational functions b from the closed unit ball of H^∞. In particular, we give a characterization of functions from local Dirichlet spaces of order m in terms of their m-th derivatives. We also find explicit formulas for b in the case when $\mathcal{H}(b)$ coincides with local Dirichlet space of order m with equality of norms.

The talk is based on joint work with B. Łanucha, M. Nowak and A. Sołtysiak

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REFLEXIVE CALKIN ALGEBRAS

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Abstract

The celebrated Argyros-Haydon space with small algebra of bounded operators initiated construction of Banach spaces with prescribed Calkin algebra (i.e. the quotient operator algebra over the ideal of compact operators), in particular of the form $C(K)$, for metrizable K, or ℓ_1. During the talk we briefly describe known results and discuss the construction of Banach spaces with Calkin algebras that are isomorphic to Banach spaces with an unconditional basis from a wide class, including ℓ_p, $1 < p < \infty$, and $L_p(0,1)$. The talk is based on a joint work with Pavlos Motakis.

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STABLE WEAK* FIXED POINT PROPERTY IN ℓ_1

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Abstract

We give several characterizations of all preduals X of ℓ_1 such that X^* has the stable weak* fixed point property for nonexpansive mappings.

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ON A CERTAIN RENORMING OF l_2

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Abstract

Inspired by[1] and [2], we are studying the existence of fixed points for nonexpansive self-mappings of weakly compact and convex subsets of l_2 renormed in such a way that a large number of known properties implying the fixed point property do not hold.

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ASTROPHYSICAL MODELS, LYAPUNOV FUNCTIONS AND OTHER APPLICATIONS

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Abstract

The existence of Lyapunov function is proved for some general dynamical system in Kolmogorov framework

$$x' = xG(x,y) \quad y' = yH(x,y)$$

with nonzero stationary solution under some monotonicity assumptions imposed on nonlinearities. The applications vary from variations of predator-prey models and epidemy equations to description of distribution of mass in astrophysics. The results were obtained in collaboration with Dorota Bors from University of Lodz and students from University of Wrocław.

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FUNCTIONS OF BOUNDED VARIATION ON REAL HALF-AXIS

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Abstract

The aim of the talk is to present some functions concerning the concept of the variation of functions on the real half-axis $\mathbb{R}_+$. We are formulating the announced definition and also some equivalent version of that definition. Moreover, we provide a few properties of functions of bounded variation on the half-axis associated mainly with the behaviour of those functions at infinity.

Apart from this we are going also to introduce the definition of the Riemann-Stieltjes integral on an unbounded interval. Some properties of such an integral will be also provided.

Our investigations will be illustrated by suitable examples.

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DARBO'S FIXED POINT THEOREM REVISITED

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Abstract

We revisit Darbo's classical fixed point theorem and propose a new extension in which the Darbo condition is required only on countable subsets. The proof integrates topological and order-theoretic techniques by jointly applying the Schauder and Knaster-Tarski fixed point theorems. The talk concludes with a discussion of several open problems.

On an oscillation criterion for second-order differential equations with deviating argument

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Abstract

As a generalization of A. Myshkis's work on differential equations with deviating argument, Marat Akhmet introduced an interesting class of differential equations of the form $x'(t) = g(t, x(t), x(\gamma(t)))$, where $\gamma(t)$ is a *generalized piecewise constant argument*. The function $\gamma(t)$ is defined as follows: Let $(t_n)_{n\in\mathbb{Z}}$ and $(\zeta_n)_{n\in\mathbb{Z}}$ be sequences satisfying $t_n \leq \zeta_n \leq t_{n+1}$ for all $n \in \mathbb{Z}$, with $\lim\limits_{n\to\pm\infty} t_n = \pm\infty$. The function $\gamma(t)$ is locally constant and defined by $\gamma(t) = \zeta_n$ for $t \in I_n = [t_n, t_{n+1})$. A fundamental example of such a function is $\gamma(t) = [t]$, where $[\cdot]$ denotes the greatest-integer function, which remains constant on each interval $[n, n+1[$ for $n \in \mathbb{Z}$.

Inspired by [1, 2, 3] and [5], we establish two Leighton-Wintner-type oscillation criteria for the following second-order differential equation with a piecewise constant argument:

$$(a(t)x(t)')' + g(t, x(\gamma(t))) = 0, \quad t \geq \tau,$$

under the same integrability conditions on the coefficients as in [3], but now with a generalized piecewise constant argument $\gamma(t)$.

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Uniqueness of Solutions for Caputo Fractional Delay Differential Equations: Progressive Contractions

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Abstract

In this study, we consider a nonlinear Caputo fractional-order differential equation with multiple variable delays. We investigate the uniqueness of solutions for this class of equations by employing the method of progressive contractions, originally introduced by T.A. Burton.

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DISCUSSION ON SHAPES OF SPHERES IN THE GENERALIZED PARANORMED HAHN SPACE

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Abstract

We introduce the generalized paranormed Hahn space $(h_d(p), g_{(p)})$, where d is an increasing unbounded sequence of positive reals, p is a sequence of positive real numbers, and $g_{(p)}$ is the paranorm on $h_d(p)$.

We apply our own software for line graphics *MV-Graphics* to visualize spheres in three–dimensional space endowed with the paranorm $g_{(p)}$. For this we give the parametric representation of parts of these spheres and solve the appropriate visibility problem.

Finally we demonstrate the effects of the change of all parameters on the shape of the spheres.

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NONLINEAR DIFFERENTIAL PROBLEMS WITH MULTIVALUED NONLINEARITIES

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Abstract

In this talk we discuss nonlinear Dirichlet problems driven by a $-\Delta_{p(\cdot)} + \mu\Delta_{q(\cdot)}$ differential operator, with variable exponents $p, q \in C(\overline{\Omega})$ where $\Omega \subset \mathbb{R}^N$ is a bounded domain with smooth boundary. First we consider the case of non-elliptic principal operator ($\mu > 0$). Using Clarke's subdifferentiability theory of locally Lipschitz functions and energy functional analysis, we prove the existence of generalized solutions. Then, we assume the principal operator is elliptic ($\mu \leq 0$). Using the theory of operators of monotone type we show that the problem has a weak solution.

The presented results are contained in Ref. [3]; the mathematical background is given in Refs. [1, 2, 4].

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THE BANACH-MAZUR DISTANCE BETWEEN $C(\Delta)$ AND $C_0(\Delta)$ EQUALS 2

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Abstract

Let $C(\Delta)$ denote the Banach space of all continuous real valued functions on the Cantor set Δ and $C_0(\Delta) = \{f \in C(\Delta) : f(1) = 0\}$. From the 1966 theorem of Cambern, it is well-known that the Banach-Mazur distance $d(C(\Delta), C_0(\Delta)) \geq 2$. We prove that, in fact, $d(C(\Delta), C_0(\Delta)) = 2$. As a consequence, we answer a question left open in the 2012 paper of Candido and Galego.

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On some class of Lipschitz mappings and their induced Markov operators

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Abstract

We consider a finite family of Lipschitz self-maps on a complete metric space X, forming an iterated function system (IFS). Based on this family, we define a Markov operator acting on the space $\mathcal{P}(X)$ of Borel probability measures on X, endowed with the Monge-Kantorovich metric d_{MK}. Under suitable conditions on the mappings, we establish the existence of an invariant Borel probability measure as a fixed point of the Markov operator. The result is obtained via a fixed point theorem applied in the complete metric space $(\mathcal{P}(X), d_{MK})$. This framework extends classical constructions in the theory of iterated function systems and provides a rigorous measure-theoretic approach to the existence of stationary measures associated with such systems.

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A FIXED POINT THEOREM FOR ISOMETRIES ON A METRIC SPACE

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Abstract

In 1955, motivated by the works of Dixmier and Day on uniformly bounded representations of groups, Kadison posed the following problem: Is every bounded homomorphism $u : A \to B(H)$ defined on a C*-algebra A similar to a $*$-homomor-phism, i.e., does there exist $S \in B(H)$ such that the map $\tilde{u}(x) = S^{-1}u(x)S$ satisfies $\tilde{u}(x^*) = \tilde{u}(x)^*$? This hypothesis is equivalent to several significant problems, including: whether every bounded homomorphism $u : A \to B(H)$ is completely bounded, whether every derivation $\delta : A \to B(H)$ on a C*-algebra $A \subset B(H)$ is inner, whether all C*-algebras have finite length, and whether all von Neumann algebras are hyperreflexive. We propose a new approach to Kadison's similarity problem based on a fixed point theorem in the space $B(H, \ell_\infty)$. More generally, we show that if X is a complete metric space with uniform relative normal structure and G is a subgroup of its isometry group with bounded orbits, then X has a point fixed by all isometries in G.

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OPERATORS WITH MEMORY IN SCHRAMM SPACES

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Abstract

An operator K acting between two classes of functions $f : I \to \mathbb{R}$, where I is a compact real interval, is called *an operator with memory* (or *locally defined*) if whenever two functions f and g from the first class coincide on some open interval $J \subset I$, their images $K(f)$ and $K(g)$ contained in the second class also coincide on J. A typical example is the Nemytskij composition operator

$$H(f)(x) = h(x, f(x)) \quad (x \in I),$$

generated by some function $h : I \times \mathbb{R} \to \mathbb{R}$.

The main aim of the lecture is to give a representation formula for operators with memory which are self-mappings of Banach spaces $C\Phi BV(I)$ of continuous functions of bounded variation in the sense of Schramm. Namely, we show that they are Nemyskij composition operators with the continuous generating function h. Moreover, under the additional assumption that the operators with memory are uniformly bounded, we observe that operators of such a type must be affine, i.e., must be of the form

$$K(f) = \alpha \cdot f + \beta,$$

where α and β are the elements from the range.

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